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## RESEARCH NOTES AND COMMENTARIES

# TOWARD A GENERAL THEORY OF COMPETITIVE DOMINANCE: COMMENTS AND EXTENSIONS ON POWELL (2003)

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*In a recent paper, Powell (2003) studied 20-year performance in 21 industries, using an ordinal performance measure ('wins'), and the Gini coefficient as a measure of competitive dominance. The findings suggest that firm performance is statistically indistinguishable from performance in non-business domains such as politics, games, sports, and pageants. This paper extends these findings, developing the statistical foundations for a general theory of competitive dominance. The paper presents a Gibrat-based null hypothesis, develops a decomposable index of competitive dominance, and suggests statistical procedures and empirical methods for future research.*


## COMPETITIVE DOMINANCE

In a recent paper, Powell (2003) examined competitive dominance in 21 industries, using as a performance measure the number of 'wins' for each firm (in profit rates, returns to shareholders, etc.) over a 20-year period (1980–99). As a statistical index, Powell computed 20-year Gini coefficients for each measure and industry.

Using wins as the performance measure enabled Powell to compare business performance with performance in games, politics, sports, and other domains. From these analyses came three conclusions: (1) performance distributions in business are statistically indistinguishable from distributions in non-business domains; (2) strategy research often uses arbitrary null hypotheses (e.g., parity) that induce excessive or inappropriate causal explanation; and (3) competitive performance can be explained 'neutrally,' without invoking player-specific advantages.[1] From these conclusions Powell argued that, rather than limiting its scope to business performance or the unique attributes of firms, a theory of competitive performance should investigate the structural and statistical

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[1] A 'neutral' theory emphasizes the structural determinants of a statistical distribution, rather than the attributes of its individual members (Hubbell, 2001).


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features common to all domains of human competition—within which firm performance arises as an instance.

This paper examines two technical aspects of the earlier paper. First, it examines the statistical foundations of Powell's 20-year performance scenario, deriving Powell's stochastic null models directly from the mathematics of random walks and Gibrat's process. Second, it compares the Gini coefficient with other measures of competitive dominance, including Herfindahl, entropy, and the likelihood ratio statistic. The analysis suggests that the latter is preferable for statistical decomposition and significance testing, and the paper shows how this statistic can be applied in future research on competitive dominance.

## THE 'WIN' SCENARIO

In a study of 20 industries, Powell (2003) found that firm 'wins' are consistent with a Pareto distribution of the form $W(r) = Cr^{-\alpha}$, where $W$ is the number of wins, $r$ is the player's rank, and $C$ is a constant. The mean coefficient $\alpha$ was identical in the industry distributions and the 107 non-business domains ($\alpha = 1.25$). The average Gini coefficients for the business and non-business distributions were 0.60 and 0.56, and their variances were identical (0.058).[2]

More than one process might produce such results, or the resemblances might be coincidental. However, the pattern suggests a common underlying process, and there is precedent in economic theory for deriving theoretical and empirical advance from broad statistical regularities. For example, the stochastic theory of market structure emerged from statistical regularities in firm size distributions (Simon, 1955; Simon and Bonini, 1958; Ijiri and Simon, 1974; Whittington, 1980; Hart and Oulton, 1996; Sutton, 1997), and increasing returns economics emerged from statistical regularities in non-diminishing economic processes (Arthur, Ermoliev, and Kaniovski, 1987; Arthur, 1989, 1994; Krugman, 1996; Pierson, 2000). These archetypes, following an empirically derived or 'retroductive' approach to theory development (Peirce, 1901; Simon, 1968), provide statistical tools for building theory and empirical work from distributional regularities, including contagion modeling, Pareto and log-normal distributions, and Gibrat's proportional growth process. We believe these tools will play an important role in future research on competitive dominance.

Powell's (2003) analysis began with the following 'win' scenario: $n$ players compete for $w$ periods, which Powell defined as 20 years. After $w$ periods, we sum the number of wins for each player and assess the statistical properties of this distribution of wins.

To illustrate the scenario, consider a simple game in which three players have competed for six periods, with 'wins' in a performance measure distributed $W = (5,1,0)$ (the first-ranked player won five times, the second won once, the third did not win). In this example, the 'perfect parity' distribution is $W_p = (2,2,2)$ and the 'total dominance' distribution is $W_d = (6,0,0)$. The initial statistical problem is twofold: (1) to find an appropriate quantity for characterizing $W = (5,1,0)$; and (2) to develop null models for empirical testing.

Under this scenario, wins accumulate sequentially, as in the 'occupancy' problem of placing $w$ objects ('wins') into $n$ baskets ('players'), one at a time (Feller, 1957; Ijiri and Simon, 1977). For a game in which three players compete for six periods, with one 'win' awarded each period, there are $n^w = 3^6 = 729$ possible assignments of wins.

However, Powell (2003) proposed a scenario in which neither the win nor the identity of the player is distinguished; i.e., a win in period two is not distinguished from a win in period five, and for any outcome such as (5,1,0) we do not distinguish which player ranks first, second or third. As such, this scenario is concerned with only seven outcomes: (6,0,0) (5,1,0) (4,2,0) (4,1,1) (3,3,0) (3,2,1) (2,2,2), where the first number represents the wins of the highest-ranked player. The problem is to characterize these outcomes statistically, to determine an appropriate null, and to evaluate the probability of obtaining any outcome.

## FINDING THE NULL MODEL

Null models should correspond with credible processes capable, at least hypothetically, of producing the observed outcomes (Ijiri and Simon, 1977; Starbuck, 1994). In the three-player

[2] The Gini coefficient ($G$) is a 0–1 measure of distributional parity. Perfect equality of outcomes yields $G = 0$, and total dominance by one competitor yields $G = 1$. The formula for computing $G$ is given later in the paper (see formula (8) and Table 3), and examples are provided in Tables 1–4.

 

example, outcome (2,2,2) seems a plausible null hypothesis, and corresponds with widely held human intuitions about economic fairness and distributive justice (Kahneman, Knetsch, and Thaler, 1986; Rabin, 1993). The problem is that no credible competitive process yields this outcome. We could contrive such a process (e.g., assigning wins by fiat to the player with the fewest wins), but such processes seldom arise in practice.

As an alternative, we consider a stochastic null, described by the general multinomial model ($M$) in which players labeled 1, 2, ..., $n$, win with probabilities $p_1$, $p_2$, ..., $p_n$. Here, the probability of any outcome ($w_1$, $w_2$, ..., $w_n$) is:

$$pr_M(W) = pr_M(w_1, w_2, \ldots, w_n) = \frac{w!}{w_1!w_2!, \ldots, w_n!} \times (p_1{}^{w_1}) \times (p_2{}^{w_2}), \ldots, (p_n{}^{w_n}) \quad (1)$$

Powell's (2003) first stochastic model assumed a random assignment of wins, each player having probability $1/n$ of winning in each period. This is the special case of (1) in which $p_1 = p_2 = \ldots p_n = 1/n$, and the multinomial probability of outcome ($w_1$, $w_2$, ..., $w_n$) is:[3]

$$pr_{MB}(W) = pr_{MB}(w_1, w_2, \ldots, w_n) = \frac{w!}{w_1!w_2!, \ldots, w_n!} n^{-w} \quad (2)$$

Applying (2) to the result (5,1,0) yields the probability 6/729. However, since the win scenario is indifferent to the orderings of firms, the probability of outcome (5,1,0) is obtained by multiplying (2) by the number of ways the numbers (5,1,0) can be permuted, which is $3! = 6$ ways.[4] Thus, the probability of $W = (5,1,0)$ is $36/729 = 0.049$.

Using this method, Table 1 shows the probabilities for all seven outcomes, along with Gini coefficients for each outcome, computed as in Powell's (2003) Appendix 1. The table shows an expected Gini of 0.39, and shows that, with wins assigned randomly, an outcome with at least as much dominance as (5,1,0) would occur roughly 5.3 percent of the time.

Random processes notoriously produce more disparity than people expect. Experiments in illusory causation show that results such as (3,2,1), (4,2,0), or (4,1,1) evoke feelings of inequality or unfairness, and prompt a search for causal explanations or remedies (Gilovich and Douglas, 1986; Keren and Lewis, 1994). However, in this example, the outcome (3,2,1) should, under random assignment of wins, occur four times as often as (2,2,2). Outcomes (4,2,0) and (4,1,1) are exactly as likely as (2,2,2), and even at $p < 0.10$, only outcomes (5,1,0) and (6,0,0) are sufficiently unlikely to justify a search for non-random causes. We return to this idea in the next section, in evaluating other stochastic nulls.

An important feature of (2) is its mathematical connection with the likelihood ratio. For example, we can measure competitive dominance by computing a ratio with (1) in the numerator, using proportions based on obtained data (such as 5/6,1/6,0), and (2) in the denominator. The numerator is the maximum likelihood probability of $W$, i.e., the probability of obtaining $W$ assuming the null probabilities matched the obtained values, e.g., $(p_1, p_2, p_3) = (5/6, 1/6, 0)$. This likelihood ratio can be expressed as follows:

$$LR = \left[\frac{pr_{\max}(W)}{pr_{MB}(W)}\right] = \prod_{i=1}^{n}\left(\frac{w_i}{w}\right)^{w_i} \Big/ \prod_{i=1}^{n}\left(\frac{1}{n}\right)^{w_i} = \prod_{i=1}^{n}\left(\frac{nw_i}{w}\right)^{w_i} \quad (3)$$

If we take twice the natural logarithm of (3), we obtain:

$$L = 2\ln LR = \sum_{i=1}^{n} 2w_i \ln\left(\frac{nw_i}{w}\right) \quad (4)$$

Expression (4) gives the *likelihood ratio statistic* ($L$), a positive number that increases as the observed win distribution diverges from a random distribution of wins. Because $L$ derives directly from the ratio of (1) and (2), it corresponds with

[3] In physics, (2) is known as the Maxwell–Boltzmann probability model (hence the MB subscript). In later sections, we compare the MB model to the Bose–Einstein (BE) probability model.

[4] In general, the number of ways three win totals can be assigned to $n$ players is $q = n!/g_1!g_2!g_3!$, where $g_i$ are the numbers of wins in each 'number group.' The number of ways the distribution (6,0,0) can be assigned is $3!/1!2!0! = 3$ (there is one 'six,' two 'zeros,' and no third value). For (5,1,0), we get $3!/1!1!1! = 6$.

Table 1. Probabilities under random assignment of wins

| Outcome | Gini | Probability (ordered) | No. of permutations | Total probability | Cumulative probability |
|---|---|---|---|---|---|
| (6,0,0) | 1.00 | 1/729 = 0.0014 | 3 | 3/729 = 0.0041 | 3/729 = 0.0041 |
| (5,1,0) | 0.83 | 6/729 = 0.0082 | 6 | 36/729 = 0.0494 | 39/729 = 0.0535 |
| (4,2,0) | 0.67 | 15/729 = 0.0206 | 6 | 90/729 = 0.1235 | 129/729 = 0.1770 |
| (4,1,1) | 0.50 | 30/729 = 0.0411 | 3 | 90/729 = 0.1235 | 219/729 = 0.3004 |
| (3,3,0) | 0.50 | 20/729 = 0.0274 | 3 | 60/729 = 0.0823 | 279/729 = 0.3827 |
| (3,2,1) | 0.33 | 60/729 = 0.0832 | 6 | 360/729 = 0.4938 | 639/729 = 0.8765 |
| (2,2,2) | 0.00 | 90/729 = 0.1235 | 1 | 90/729 = 0.1235 | 729/729 = 1.0000 |
| Total | | | 28 | 1 | |

Notes and assumptions:
1. Three players ($n = 3$); six wins ($w = 6$).
2. Assumes each player has probability of winning 1/3 in each period (probabilities determined from Equation 2 in text).
3. Probability (ordered) is the probability of any outcome; e.g., (6,0,0) for players (1,2,3), in that order.
4. No. of permutations: the number of ways (orders) a given outcome can occur, e.g., (6,0,0) (0,6,0) (0,0,6).
5. Total probability is the probability of any outcome—say, (6,0,0)—in any order.

the original win scenario, and can be regarded as an index of competitive dominance under the random assignment (MB) null.

We also note that $L$ relates closely to another well-known statistic, Pearson's *goodness of fit* (Pearson, 1900), which can be expressed:

$$P = nw^{-1}\sum_{i=1}^{n}\left(w_i - \frac{w}{n}\right)^2 \qquad (5)$$

Under the random assignment (MB) null, both $L$ and $P$ have approximate chi-square distributions with $n - 1$ degrees of freedom. Further, Lloyd (1999: 93) has shown that $L$ and $P$ are numerically similar except under extreme violations of the random assignment null. In the next section we show that, in the basic (non-decomposed) scenario, $L$ produces results ($p$-values) identical to entropy, and $P$ produces results identical to Herfindahl.

## ALTERNATIVES TO THE STANDARD NULL

Is a random walk the appropriate null model for firm performance? Its appeal stems from linkages with perfectly competitive product and factor markets. But Powell argues that different firms cannot be homogeneous, either theoretically or empirically (see also Powell, 2001, 2002). If firms differ, they differ in their attributes. Unless a performance measure handicaps or randomizes these differences—as in horse-racing or coin toss—then equal probability is not a credible null. A truly strong-form efficient share market would effectively handicap risk-adjusted shareholder returns, but no credible process produces a comparable effect for accounting profits, profit rates, or revenue growth.[5] If players compete in a multi-period game without randomization or handicaps, then a random walk cannot describe the outcome. For this reason, Powell (2003) proposed alternative null models using baselines from other competitive domains.

In Powell's data, 40 of 42 profit rates, and 92 of 107 non-business performance measures, produced positive $z$-scores relative to random assignment. The mean Gini coefficient for all competitive domains was 0.58, compared with 0.24 under random assignment. For business strategy research, this suggests two possibilities: (1) the random walk is a credible null, and firm-specific advantages are widespread; or (2) competitive performance is inherently non-random, and we are using the wrong null model. Whereas the former view induces a theory of unique, player-specific causes, the latter suggests a 'neutral' approach that identifies underlying performance generation

[5] Standard microeconomic theory provides a logic for the equalization of economic profits, but there is no reason to expect this to apply to accounting profits, or to any sector with imperfectly competitive product or factor markets. Also, Lippman and Rumelt (1982) and others have shown that, under uncertainty, equality of economic profits is not to be expected even under free entry and atomistic price-taking.

 

processes capable of producing the observed distributions, within which business performance may be regarded as an instance.

The latter view requires us to explain the evolutionary processes through which long-term performance distributions emerge, and this is the leading task for future research on competitive dominance. At present, what arises from the data is a statistical picture consistent with Gibrat's (1931) proportional growth heuristic. Applied to the win scenario, 'Gibrat's law' proposes that, given a basic performance history, each firm's probability of winning in the current period is proportional to its past wins. It has been shown elsewhere that this process generates log-normal Pareto distributions consistent with the findings in Powell (2003).[6] It can also be shown that a Gibrat process produces distributions identical to a model in which each *outcome* has equal probability, rather than each player having equal probability of winning in each period (Ijiri and Simon, 1977).[7]

This model, known as Bose–Einstein (BE) statistics, can be illustrated using the earlier example. In Table 1, to compute the total number of BE outcomes, we distinguish one competitor from another, e.g., we distinguish the outcome $(w_1, w_2, w_3) = (5,1,0)$ from $(w_1, w_2, w_3) = (1,0,5)$. In general, the number of outcomes that distinguish players but not wins is:

$$\text{Outcomes}_{\text{BE}} = \frac{(n+w-1)!}{(n-1)!w!} \tag{6}$$

In the Table 1 example, this number is 28 outcomes; i.e., if three players compete for six periods, there are 28 possible outcomes that distinguish players but not wins. These 28 outcomes are listed in Table 2.

In the BE model, each of these outcomes occurs with equal probability, so that the BE probability for any outcome is:[8]

$$\text{pr}_{\text{BE}} = \frac{(n-1)!w!}{(n+w-1)!} \tag{7}$$

Applying this model to our example, each outcome has probability 1/28. We then collapse the 28 BE outcomes into the seven outcomes of interest in the original win scenario, yielding the results shown in column BE. Using BE probabilities, total dominance (6,0,0) has probability 3/28, perfect parity (2,2,2) has probability 1/28, outcome (5,1,0) has probability 6/28, and the expected Gini coefficient is 0.60. In general, the BE probabilities predict greater inequality than the MB model, and the model can be generated analytically or through simulations using an underlying Gibrat or Polya process.

We believe a contagion process, rather than a random walk, is the correct underlying process for understanding the emergence of competitive dominance, and we propose BE probabilities, rather than MB probabilities, as the statistical null model. Gibrat's process generates outcomes consistent with Powell's findings, and it has mathematical linkages with the likelihood ratio statistic, entropy, log-normal Pareto distributions, Polya's urn, and Markov processes.[9]

## STATISTICAL MEASURES OF COMPETITIVE DOMINANCE COMPARED

Under the win scenario, we could describe competitive dominance using equality indices (such as the Gini coefficient) or measures of concentration (such as four-player concentration, Herfindahl, or

[6] A log-normal distribution has a long right tail, and is equivalent to transforming $x$ values in a normal distribution using $e^x$. In Gibrat's (1931) original research, plotting firm size intervals on the $x$-axis, and numbers of firms per interval on the $y$-axis, gave a log-normal distribution. Using the same data, plotting size *ranks* on the $x$-axis, and sizes on the $y$-axis, gives a Pareto distribution (see Simon, 1968, 1993).

[7] In contagion modeling, 'Polya's urn' produces equivalent outcomes, and has been used extensively in increasing returns economics (see Arthur *et al.*, 1987). In brief, we imagine an urn containing one white ball and one red ball. We draw a ball at random, note the color, and replace it in the urn, along with another of the same color drawn. If the color drawn is red, we replace the red and add another red, and the probability of choosing red in the next period is 2/3. If three firms compete for six periods according to Gibrat's heuristic, then we imagine six successive draws from Polya's urn containing balls of three colors. After six draws, we count the number of drawn balls of each color ('wins'), and this constitutes the performance distribution.

[8] In particle physics, it was found that distributions of photons do not obey MB probabilities. Bose and Einstein proposed Model 7, in which each distinguishable outcome has equal probability.

[9] This does not demonstrate the economic significance of Gibrat's process, i.e., why competitive performance evolves along lines consistent with a contagion model. This issue was addressed in the concluding sections of Powell (2003), and requires development in future research. One conjecture is that performance emerges from strategic capital accumulation processes, following trajectories analogous to learning or experience curves, which can in turn be described as distributed statistical aggregates (rather than as player-specific attributes).

Table 2. Comparing Bose–Einstein and Maxwell–Boltzmann probabilities

| Outcome | Gini | Probability (ordered) | No. of permutations | Permutations | Total BE probability | Total MB probability |
|---|---|---|---|---|---|---|
| (6,0,0) | 1.00 | 1/28 = 0.0357 | 3 | (6,0,0) (0,6,0) (0,0,6) | 3/28 = 0.1071 | 3/729 = 0.0041 |
| (5,1,0) | 0.83 | 1/28 = 0.0357 | 6 | (5,1,0) (5,0,1) (1,5,0) (1,0,5) (0,5,1) (0,1,5) | 6/28 = 0.2143 | 36/729 = 0.0494 |
| (4,2,0) | 0.67 | 1/28 = 0.0357 | 6 | (4,2,0) (4,0,2) (2,4,0) (2,0,4) (0,4,2) (0,2,4) | 6/28 = 0.2143 | 90/729 = 0.1235 |
| (4,1,1) | 0.50 | 1/28 = 0.0357 | 3 | (4,1,1) (1,4,1) (1,1,4) | 3/28 = 0.1071 | 90/729 = 0.1235 |
| (3,3,0) | 0.50 | 1/28 = 0.0357 | 3 | (3,3,0) (3,0,3) (0,3,3) | 3/28 = 0.1071 | 60/729 = 0.0823 |
| (3,2,1) | 0.33 | 1/28 = 0.0357 | 6 | (3,2,1) (3,1,2) (2,3,1) (2,1,3) (1,3,2) (1,2,3) | 6/28 = 0.2143 | 360/729 = 0.4938 |
| (2,2,2) | 0.00 | 1/28 = 0.0357 | 1 | (2,2,2) | 1/28 = 0.0357 | 90/729 = 0.1235 |
| Total | | | 28 | | | |
| Expected Gini | | | | | Gini = 0.60 | Gini = 0.39 |

Notes and assumptions:
1. Three players ($n = 3$); six wins ($w = 6$).
2. BE model assumes each permutation is equally probable (ordered probability is 1/28 for every ordered outcome) (probabilities determined from Equation 7 in text).
3. MB probabilities taken from Table 1.

entropy). The Gini coefficient is widely used but, as shown earlier, the likelihood ratio statistic ($L$) and Pearson's goodness-of-fit statistic ($P$) have direct statistical relations with the win scenario. For empirical research, we believe $L$ provides more convenient statistical tests, more reliable statistical decomposition, and better adaptability to alternative null models.

Algebraically, the Gini can be expressed in various ways (Gastwirth, 1971, 1972; White, 1981; Atkinson, 1970, 1983; Thistle, 1990; Zheng and Cushing, 2001). Where $n$ is the number of players, and $w_i$ the number of wins for the $i$th-ranked player, one formulation is:

$$G = \frac{w(n+1) - 2\left(\sum_{i=1}^{n} i w_i\right)}{w(n-1)} \tag{8}$$

The Gini coefficient does not have a simply expressed approximate distribution under random assignment or Gibrat's process, or when $n$ and $w$ are large (see Chernoff, Gastwirth, and Johns, 1967). Powell avoided this problem by using simulations, but in some empirical projects researchers will prefer indices with known distributions (such as chi-square), or that decompose under convenient assumptions. Here, we evaluate alternative measures.

Table 3 presents five disparity indices: $G$, $E$, $H$, $L$, and $P$. Table 3 also gives the statistical distributions for each, and applies them to the simple numerical example (5,1,0) from the previous section. The following sections evaluate three properties of these indices: ease of significance testing, decomposition properties, and adaptability for testing Bose–Einstein and other null models.

## Ease of significance testing

The Herfindahl index is a well-known measure of market concentration (Hirschman, 1964; Schmalansee, 1977; Scherer and Ross, 1990). In the present context, $H$ can be interpreted either as a direct index of competitive dominance, or as the probability that the same firm will win in two consecutive years. As such, $H$ becomes smaller as $n$ rises, and its inverse ($1/H$) has been interpreted as a 'numbers equivalent,' i.e., the number of equally performing competitors equivalent to the observed

Table 3. Comparing measures of competitive dominance

| Measure | Statistic | Value under perfect parity | Value under total dominance | Significance tests | Value for example in text |
|---|---|---|---|---|---|
| Gini | $G = \frac{(n+1) - 2\left(\sum_{i=1}^{n} i q_i\right)}{(n-1)}$ | 0 | 1 | Simulation | $G = 0.83$ ($p = 0.053$) |
| Entropy | $E = -\sum_{i=1}^{n} q_i \ln q_i$ | ln $n$ | ln 1 = 0 | $\chi^2$ (convert to $L$) | $E = 0.45$ ($p = 0.022$) |
| Herfindahl | $H = \sum_{i=1}^{n} q_i^2$ | $1/n$ | 1 | $\chi^2$ (convert to $P$) | $H = 0.72$ ($p = 0.032$) |
| Pearson's goodness-of-fit | $P = wn \sum_{i=1}^{n} \left(q_i - \frac{1}{n}\right)^2$ | 0 | $w(n-1)$ | $\chi^2$ (d.f. $= n-1$) | $P = 7.00$ ($p = 0.032$) |
| Likelihood ratio | $L = 2w\left(\sum_{i=1}^{n} q_i \ln nq_i\right)$ | 0 | $2w(\ln n)$ | $\chi^2$ (d.f. $= n-1$) | $L = 7.78$ ($p = 0.022$) |

Notes:
1. Notation: $n$ = number of players; $w$ = number of wins; $q_i$ = proportion of wins for $i$th-ranked player.
2. Example in text: $n = 3$; $w = 6$; outcome is $W = (5,0,1)$.
3. Numbers equivalents (see text): In the example, $1/H = 1.39$; $e^E = 1.57$ (Theil's entropy measure).
4. The above formulas are equivalent to those presented in the text, but for computational convenience we have substituted proportion of wins ($q_i$) for the $p_i$ and $w_i$ terms used in the text.

distribution (Adelman, 1969; see also Table 3).[10] $H$ is also equal, under Nash equilibrium, to the Lerner index of monopoly power (Hause, 1977).

As noted earlier, $H$ produces $p$-values identical to those of the Pearson goodness-of-fit statistic ($P$). By the following relation, the Herfindahl index is easily converted to $P$, and tested using a chi-square test with $n - 1$ degrees of freedom:

$$P = w(nH - 1) \quad (9)$$

Entropy, a statistical measure of systemic disorder, was suggested in its present form by Shannon (1949), and has been widely used as a measure of variability in strategy, economics and sociology (Jacquemin and Berry, 1979; Conceicao and Galbraith, 2000; Coleman, 1964). Theil (1967) also proposed the quantity $e^E$ as a numbers equivalent, similar in concept (though not equivalent) to $1/H$. As $H$ is convertible to $P$, entropy ($E$) is convertible to the likelihood ratio statistic ($L$), as follows:

$$L = 2w(\ln n - E) \quad (10)$$

We noted earlier that the likelihood ratio statistic and Pearson's goodness-of-fit statistic produce related, but not identical, statistical tests. By (9) and (10), $H$ and $E$ are related by the following approximation:

$$H \approx \frac{1 + 2\ln n}{n} - \frac{2}{n}E \quad (11)$$

In sum, whereas the Gini is incongenial to significance testing, $P$ and $L$ relate naturally to the win scenario, and can be tested using chi-square on $n - 1$ degrees of freedom.

## Decomposition properties

Future research will require measures of within- and between-group competitive dominance (e.g., to examine differences across industries or market segments). This form of decomposition was addressed in a general setting by Cowell (1980), who concluded from plausible axioms that only entropy indices decompose viably for the study of income inequality. In strategy, researchers have investigated the decomposition properties of entropy and other indices as measures of concentration (Acar and Sankaran, 1999) and corporate diversification (Jacquemin and Berry, 1979; Palepu, 1985; Hoskisson *et al*., 1993; Hall and St. John, 1994; Raghunathan, 1995; Hoopes, 1999; Robins and Wiersema, 2003). Here, we investigate whether any of the five measures produce useful decompositions under the win scenario.

We find that, of the five measures, only the likelihood ratio statistic ($L$) decomposes in a clearly interpretable way. This is shown in the example in Table 4. Eight firms compete for 20 years, and the distribution of wins is (7,4,3,3,1,1,1,0). We segment the firms into three groups —A, B, C—for which the distributions of wins are, respectively, (4,3,3), (1,1), and (7,1,0). The problem is to characterize within-group, between-group, and overall dominance.

Table 4 shows within-group, between group and overall dominance for each of the five measures. $L$ is large overall ($L = 14.865$; $p = 0.038$ on d.f. $= 7$), and there is dominance within C ($p = 0.002$ on d.f. $= 2$), but not within A or B. Clearly, $L_A$, $L_B$, and $L_C$ do not sum to 14.865, but 11.743. This is true of all measures, but for $L$ the difference $14.865 - 11.743 = 3.121$ is itself a likelihood ratio statistic, and can be interpreted as between-group dominance, with d.f. $=$ (number of groups $- 1$). In the example, between-group $L$ is not significant ($p = 0.20$ on d.f. $= 2$). Thus, we find that dominance is significant overall, and significant in group C, but not in groups A or B, or between groups.

It can be shown that all five measures decompose (see Yntema, 1933; Hall and St. John, 1994), but that only $L$ decomposes without resort to *ad hoc* weightings for the between-group test.[11] In the example, group B had one fewer firm than groups A and C, and thus would not be expected to have equal wins, even under parity. Of the five measures, only $L$ copes effectively with differently sized groups by adjusting for group sizes in the between-group test. Indeed, when group sizes are unequal it is not clear how any other statistic could meaningfully decompose under the win scenario.

[10] Adelman also points out the relation $H = n\sigma^2 + \mu$, where $\mu = 1/n$ (the mean share for each player), and $\sigma$ is the standard deviation of the shares.

[11] A general proof is available from the authors.

Table 4. Decomposing five measures of competitive dominance

| | Group A | Group B | Group C | Industry total | Between-groups | Weightings for A; B; C |
|---|---|---|---|---|---|---|
| Number of firms ($n$) | 3 | 2 | 3 | 8 | | |
| Total wins by group ($w$) | 10 | 2 | 8 | 20 | | |
| Win distribution by group | (4,3,3) | (1,1) | (7,1,0) | (7,4,3,3,1,1,1,0) | | |
| *Analysis by index* | | | | | | |
| $G$ (Gini) | 0.100 | 0.000 | 0.875* | 0.514* | 0.400 | 0.25; 0.01; 0.16 |
| $E$ (Entropy) | 1.089 | 0.693 | 0.377* | 1.708* | 0.943 | 0.50; 0.10; 0.40 |
| $H$ (Herfindahl) | 0.340 | 0.500 | 0.781* | 0.215* | 0.420 | 0.25; 0.01; 0.16 |
| $P$ (Pearson) | 0.200 | 0.000 | 10.750* | 14.400* | 2.667 | 1.333; 0.400; 1.067 |
| $L$ (Likelihood) | 0.194 | 0.000 | 11.549* | 14.865* | 3.121 | 3/8; 2/8; 3/8 |

* Significant at $p < 0.05$.
Notes and assumptions:
1. Eight firms compete in one industry for 20 years.
2. Firms segmented into three groups (A, B, C).
3. Competitive dominance computed using formulas in Table 3.
4. Industry numbers equivalents: $1/H = 4.54$; $e^E = 5.53$

## CONCLUSIONS

This paper extends the analysis in Powell (2003), and develops tools and methods for a general theory of competitive dominance. In sum, we find that:

1. Because the win scenario derives from multinomial probabilities, it links more naturally with the likelihood ratio than with the Gini coefficient.
2. Powell's (2003) empirical results are consistent with underlying Gibrat processes, which in turn can be represented using Bose–Einstein probabilities.
3. The likelihood ratio statistic enables the most complete analysis of win scenarios, whether under the random assignment null, a Gibrat process (the Bose–Einstein null), or statistical decomposition.

These extensions establish the mathematical foundations of the win scenario, and provide statistical tools for future research. Using $L$, the empirical method is much enriched, providing for wide variations in parameters $n$ and $w$, chi-square testing against a range of plausible null models (including Bose–Einstein), and statistical decomposition of within- and between-group competitive dominance. We believe these are essential tools for any future development of this research.

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